Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 30515

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to: All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. Find the third eigen value and also find the product of eigen values of A.

2. Write the quadratic form corresponding to the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & -4 \\ 3 & -4 & -3 \end{bmatrix}$.

3. Find the domain of the function $f(x) = \dfrac{1}{x^2 - x}$.

4. Prove that $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ does not exist.

5. Find $\dfrac{dy}{dx}$, if $x^3 + y^3 = 3axy$.

6. If $u = \dfrac{y^2}{x}$ and $v = \dfrac{x^2}{y}$, then find the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$.

7. Evaluate $\int \dfrac{\cos\theta}{\sin^3\theta} d\theta$ by the method of substitution.

8. Determine the following integral is convergent or divergent. $\int_0^\infty e^x dx$.

9. Evaluate $\int_1^2 \int_2^5 [xy] dxdy$.

10. Find the limits of the integration $\iint_R f(x,y) dxdy$ where R is the region bounded by the lines $x = 0$, $y = 0$ and $x + y = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ to the canonical form through an orthogonal transformation. (16)

12. (a) (i) Find the equation of the tangent line to the curve $y = \frac{e^x}{(1+x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and minimum values of the function $f(x) = \log(x^2 + x + 1)$ in $[-1, 1]$. (8)

Or

(b) (i) Show that the function $f(x)$ is continuous on $(-\infty, \infty)$ $f(x) = \begin{cases} 1 - x^2; & x \le 1 \\ \log x; & x \ge 1 \end{cases}$ (8)

(ii) Find the local maxima and minima for the function of the curve $y = x^4 - 4x^3$. (8)

13. (a) (i) If $u = \sin^{-1}\left[\frac{x^3 - y^3}{x + y}\right]$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. (8)

(ii) Find the maximum and minimum values of $f(x, y) = x^2 - xy + y^2 - 2x + y$. (8)

Or

(b) (i) Using Taylor's series, expand $f(x, y) = x^2 y + \sin y + e^x$ upto the second degree terms at the point $(1, \pi)$. (8)

(ii) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring the least material for its construction. (8)

14. (a) (i) Evaluate $\int x^2 e^x dx$ by using integration by parts. (8)

(ii) Evaluate $\int \frac{dx}{\sqrt{3x^2 + x - 2}}$ (8)

Or

(b) (i) Evaluate $\int \frac{x + 4}{6x - 7 - x^2} dx$. (8)

(ii) Evaluate $\int_{-\pi/4}^{\pi/4} [\tan^2 x \sec^2 x] dx$. (8)

15. (a) (i) Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and hence evaluate it. (8)

(ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. (8)

Or

(b) (i) Evaluate $\iint (x^2 y + xy^2)\, dx dy$ over the area between $y = x^2$ and $y = x$. (8)

(ii) Evaluate $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} [z]\, dz\, dy\, dx$. (8)
